EK-9

Lab Notes:

Shepard Functions

By: John S. Simonton, Jr.

The first audio illusion I ever heard was the Shepard Tone. Maybe you know it by the more descriptive term "barber pole" tone. It got that name because, like the stripes on a barberpole, it seems to defy the old saw "what goes up must come down". The effect is that of a continuously rising (or falling) tone which never resolves.

How the Shepard Tone works. There is nothing very mysterious about the Shepard Tone, as disconcerting as it can be at first, and if you've worked with synthesizers for a while you can figure out pretty quickly what's going on. The spectrum of the tone consists of a large number of octavely related components, all stepping up-scale together. The harmonics at the high and low ends of the spectrum have relatively low amplitudes, while harmonics in the middle of the tone are at maximum amplitude.

Imagine for a moment that you are following the lowest harmonic that makes up the Tone. At first the amplitude of this component is so low that it is, for all practical purposes, inaudible. But as it steps up-scale its level increases, peaking when its frequency corresponds to a point midway between the high and low limits. After peaking, the amplitude decreases as the frequency continues to step higher until finally, at the upper frequency limit, the harmonic is again inaudible.

When the harmonic reaches the high frequency limit it disappears, only to be replaced by a new harmonic at the lower limit.

Since the eight or ten harmonics which make up the Tone are all rising in a "staggered" progression, each in turn starting over again as it reaches its upper frequency limit, the overall effect is that of a tone which is constantly increasing in pitch while not actually getting any "higher" (or lower if the tones are all falling). It's an interesting illusion.

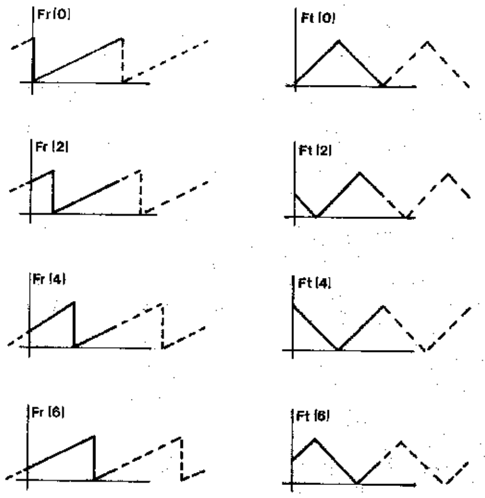
Shepard's original work used a computer program written by Max Matthews, but the same type of effect can be accomplished using analog synthesis equipment controlled by a gadget which, for lack of a better name, we may as well call a "Shepard Function Generator".

The Shepard Function Generator. Thinking about what happens with the frequency and amplitude of each harmonic of a Shepard Tone makes it easier to understand the composite sound. The frequency increases constantly and linearly from low to high, until the higher limit is reached. At this point, it begins again at the lower limit. This is a ramp function. The amplitude of any harmonic increases from the lower limit until it reaches the middle frequency, and then decreases as it approaches the upper limit. This can be a triangle function.

So, we need a gizmo which will produce a bunch of ramp waveforms (eight is a convenient number), and an equal number of triangle waves. The ramps and triangles both must have fairly precise phase relationships to one another, as summarized in the diagram in figure 1.

In the interest of conserving drawing time and space, I have shown only four of the eight functions pairs that our device will generate. I'm sure that you can see the pattern, and that the missing odd functions fit between the even functions that are shown. Notice that each function pair in the complete series is 45 degrees (pi/4 radians) out of phase with each of its neighbors. The even pairs shown are 90 degrees out of phase with one another.

Now, there are almost certainly lots of possible analog ways to generate these function



SHEPARD FUNCTION PHASE RELATIONSHIPS

figure 1

pairs. But the simplest circuits I can think of to do this are too simple (for example, they wouldn't be able to generate the functions over a very wide frequency range), and what complicated approaches come to mind are very complicated. But I know of some workable digital approaches and I'd like to show you one, a discrete logic machine.

The day will soon be here when we wouldn't even discuss a logic machine approach to this problem. We would just truck ourselves down to our local electronics store and pick up a blister packed Single Chip Data Processor to be programmed on our trusty Home Data System.

No doubt, but let's look at a way to do essentially the same thing with counters, DACs, MUXes, and such...things we can get today. Figure 2 shows some components which we'll use in the Shepard Function Generator (as you probably realize, a counter connected directly to a DAC generates ramps). If the counter is counting up, upward sloping ramps come out. Having the counter count down, or inverting the counter output before it gets to the DAC, produces downward ramps (see figure 3).

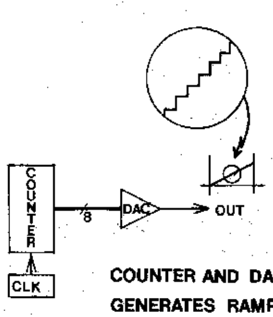
Consider this: A triangle may be thought of as a ramp which changes its mind halfway up. If we replace the inverters in the figure above with Exclusive-OR gates, we can produce a single logic input that when high, causes the DAC to produce an upward ramp and when low, causes a downward ramp. By using the most significant bit of the counter as well as the control signal to the EX-ORs, the digital input to the DAC will count up until the MSB goes high, then it will count down -- in other words, a triangle function (see figure 4).

If we're interested in generating only a single function pair, it's a simple matter to pick up a new Least Significant Bit on the counter and use it to effectively switch back and forth between the circuitry of figures 3 and 4, producing first a small section of the ramp, and then a small section of the triangle. This new LSB also switches between two sample-and-hold circuits to de-multiplex the composite output of the DAC. Figure 5 shows how a little more logic gives Fr(0) and Ft(0).

I'm sure that we're together so far, and to make sure that we stay together I should mention a useful way to think of the eight Most Significant Bits of the counters. Think of them as phase, summarized in Table 1 below.

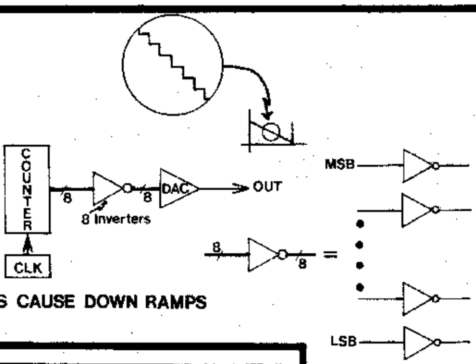
TABLE 1

Counter Output binary	hex	Equivalent Phase
00000000 -	$00	0 degrees
00100000 -	$20	45 degrees
01000000 -	$40	90 degrees
01100000 -	$60	135 degrees
10000000 -	$80	180 degrees
10100000 -	$A0	225 degrees
11000000 -	$C0	270 degrees
11100000 -	$E0	315 degrees



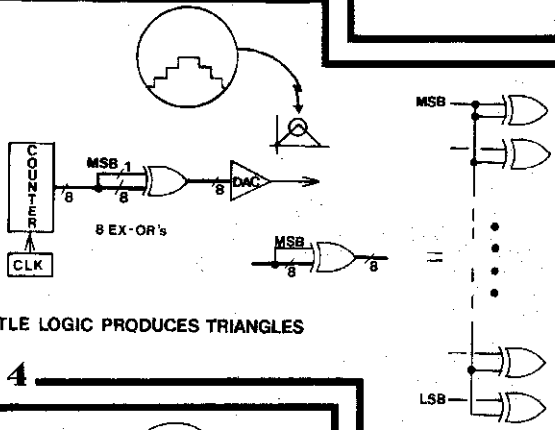
COUNTER AND DAC GENERATES RAMPS

figure 2



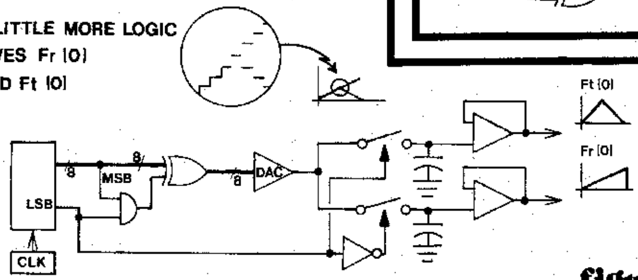
INVERTERS CAUSE DOWN RAMPS

figure 3



A LITTLE LOGIC PRODUCES TRIANGLES

figure 4



A LITTLE MORE LOGIC GIVES Fr (0) AND Ft (0)

figure 5

If you're more comfortable with a graphic representation, see figure 6. The benefit of thinking of the counter data in this way is that phase shifts are produced by simple additions. For example, to shift the phase of the waveforms produced by the counter and DAC by 45 degrees, simply add $20 to the output of the counter. This is a pretty handy thing to know, particularly when it just happens that we are looking for a way to generate eight sets of functions which are 45 degrees apart.

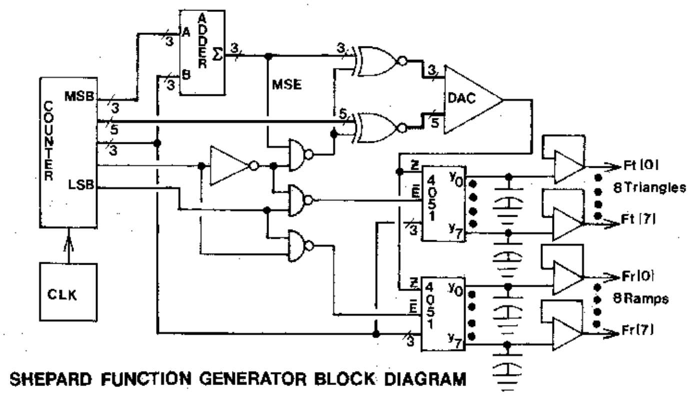
Figure 7 shows a block diagram of the complete Eight Phase Shepard Function Generator which results when we include an added IC to calculate digital phase offsets and de-multiplex the output with 8/1 analog switch ICs; figure 8 shows the schematic for the complete Shepard Function Generator. In the same way that the circuit of figure 5 alternately generated pieces of Fr(0) and Ft(0), the Shepard Function Generator sequentially puts out pieces of Fr(0), Ft(0), Fr(1), Ft(1), Fr(2).......Ft(6), Fr(7), Ft(7).

Details: Starting from the Most Significant end of the counter, the first eight bits of the counter serve the same functions that they did in the warm-ups. And we've decided to think of that function as phase. Unlike the previous sketches, these phase bits are broken down into two groups: the three Most Significant and the next five. If you don't see the significance of this grouping, review the binary representation in Table 1. To produce 45 degree phase shifts, the three Most Significant Bits are the only ones which change.

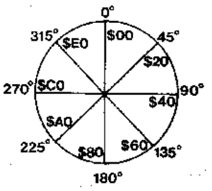
Below the eight phase bits, you'll see another grouping of three bits. Think of these as "offset" bits and notice that they are what's added to the three Most Significant Bits by the adder. And note that the offset bits also serve as address bits for the De-Muxes so that any given phase offset always gets strobed into the same sample-and-hold.

The next "Less Significant" bit can be thought of as switching back and forth between ramps and triangles, as in figure 5. And since figure 7 is a block diagram of our working Shepard Function Generator, and not just theoretical like the previous figures, the Least Significant Bit of the counter serves as a strobe which allows time for the DAC to settle

figure 7

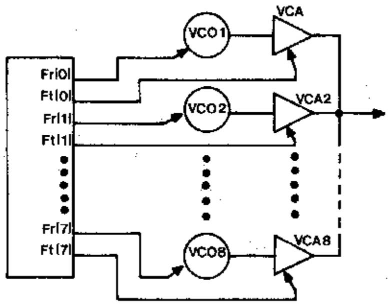


SHEPARD FUNCTION GENERATOR BLOCK DIAGRAM



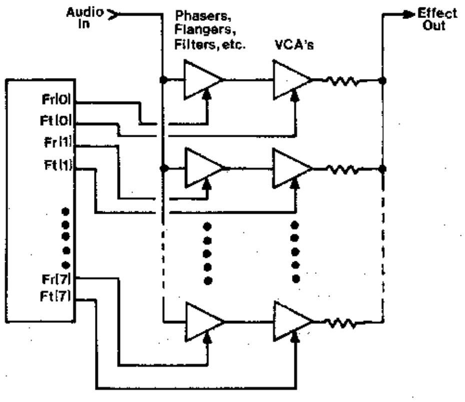
PHASE TO 8BIT BINARY RELATIONSHIPS

figure 6



VCO's and VCA's controlled by SFG produces shepard tones.

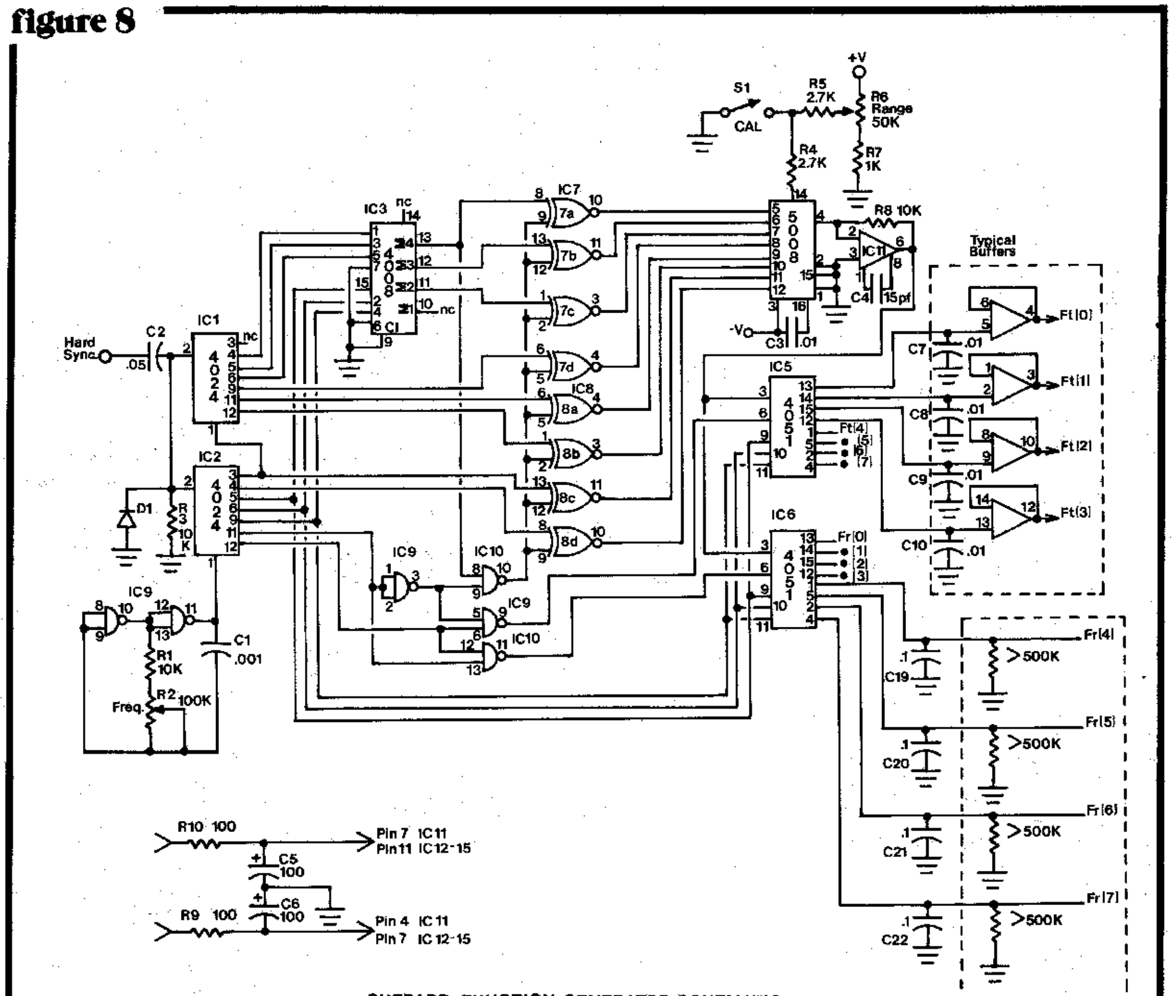
figure 9



Signal processing elements may be substituted for VCO's to produce interesting effects.

figure 10

figure 8



SHEPARD FUNCTION GENERATOR SCHEMATIC

before selecting a sample-and-hold.

So, the Shepard Function Generator which we've developed isn't simple (though I would like to think it has a certain elegance) and when you consider that we'll also need eight VCOs and eight VCAs to produce the Tone (see figure 9), you might question if it's really worth the hassle.

But Shepard Tone generation is not the only application for this circuit; recently the same principles have been applied to other areas of electronic music. For example, the Barberpole Phaser invented by Harald Bode is a signal processing device which substitute phase change components applied to an external signal source for the frequency components of the Tone. The characteristic of multiple phase shifters are controlled by Shepard Functions so that the phasing effect doesn't simply swing back and forth, like we're used to hearing, but rather sweeps up and down eternally. It's really a most unusual effect and if it has occurred to you that the same principle might also work with other processing elements (such as filters, maybe) you're on the right track.

Figure 10 is the configuration such approaches would customarily take, with the ramp functions controlling the parameter being modified (phase, corner frequency, time delay, etc.) and the triangles controlling VCAs to fade the output of each modifier in and out.

You may have noticed that we're using gobs of equipment...lots of phasers or flangers or whatever. Chances are that you don't have eight flangers laying around. Even if you us the least expensive modifiers available (PAIA's EKx module series, for example) you will still have some bucks tied up in repetitive elements. For those who lament the fact that there don't seem to be any new effects, this one qualifies. It's unique all right, but worth the cost...?

Wait. We're being prejudiced by what we've seen so far (always a danger). We're thinking of the Shepard Function Generator only as a way to generate monophonic, noncyclic illusions by always using all 16 output functions to control 16 corresponding processing elements. But that's where we're getting of the track: You don't have to use all the outputs all the time, and the results don't have to be monophonic.

Now, there's no doubt that eight phase Shepard Functions are the absolute minimum number of components which will still preserve the "barberpole" illusion, but there are other times when sets of phase synchronized functions are useful. Is it obvious that any pair of triangles 180 degrees apart -- Ft(0) and Ft(4), for instance -- may be used with a pair of VCAs to give automatic stereo panning? Or that four triangles 90 degrees apart provide quad panning? With the arrangement shown in figure 11, the apparent "revolution" of the sound source is clockwise. To reverse the apparent direction, reverse either pair of corner sources.

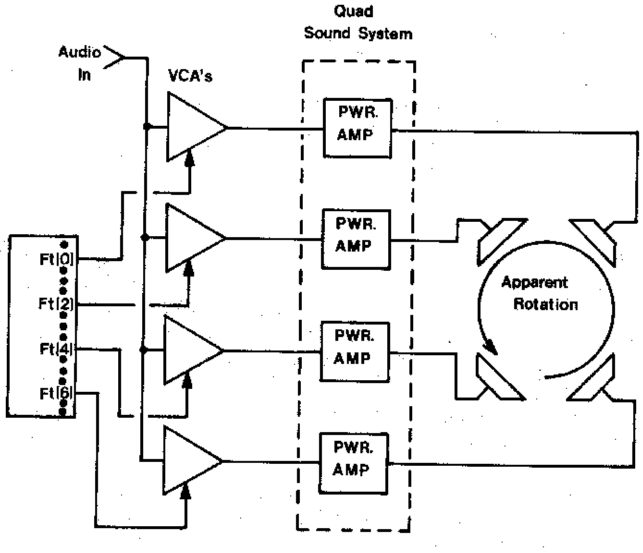
Various combinations of triangles with unequal phase relationships may be used to produce effects which don't just swing round and round, but rush out of one of the "corners", swing around in front of (or behind) you to disappear into the other corner. When you start adding effects into this setup (such as phase shifters) under control of the ramps, as shown in figure 12, the sound really begins to move around you in some strange ways.

A nice thing about this is that the effects devices don't all have to be the same to produce interesting results. In fact, some of the most interesting results come from using completely different effects (such as phaser and echo) in opposite corners with only VCA processing on the other corners. While you might be hesitant to rush out and buy eight VCOs just to get a tone, you probably have enough modules or effects to get started. Voltage control is obviously preferable, but even effects which have only manual control are useful. Among other things, be sure to try synchronizing the frequency of the effects oscillator to the frequency of the Shepard Function Generator.

I think you get the idea: Play. Try different effects and different functions applied to different effects. Try controlling the VCAs with the ramps and the effects with the triangles -- try leaving out the VCAs altogether. Not all of the results will be particularly pleasant, but you will surely also find some that are unique beyond words. While many of these effects are somewhat less spectacular when done in stereo, they are still very effective.

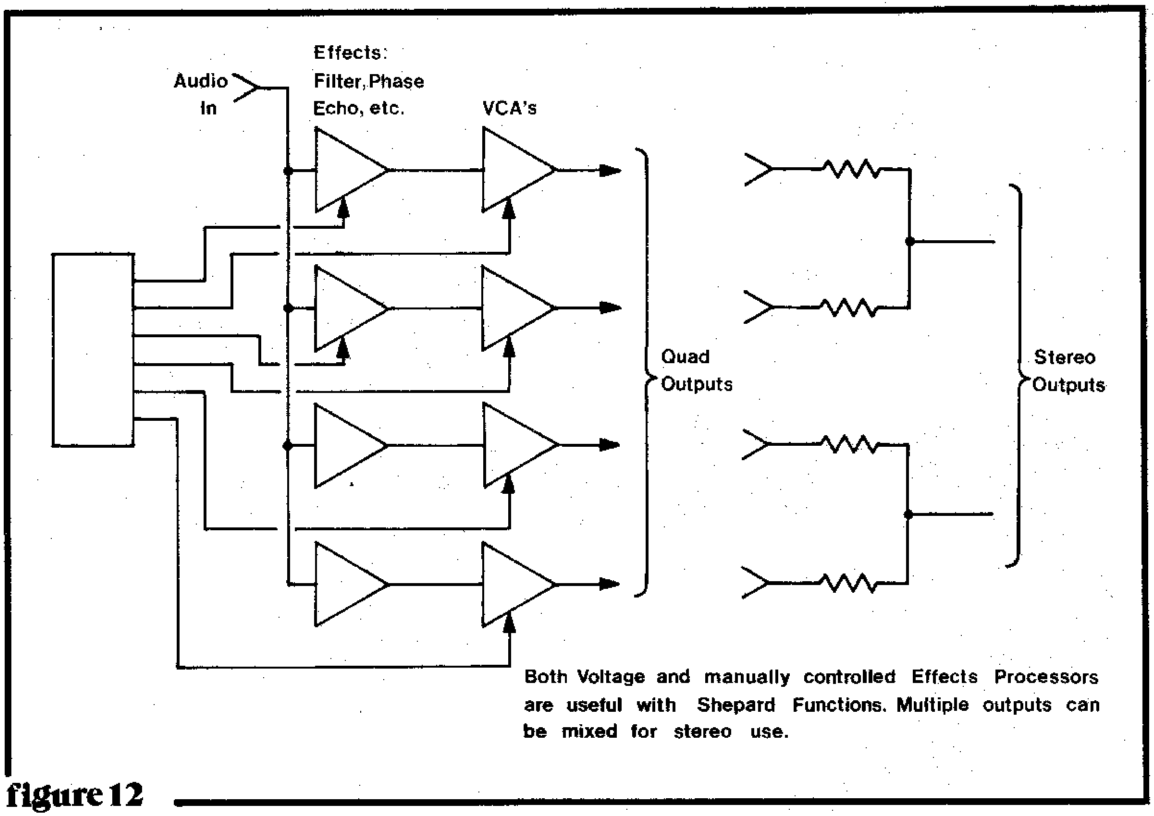
This is getting long just when I could go on forever; but it has to end as soon as I draw your attention to the hard sync input. A positive pulse applied to this input resets the counter chain and causes all functions to start from the same known point. This feature will be particularly useful to us as Craig Anderton introduces us to Synchro-Sonic techniques in future issues of Polyphony.

No new effects indeed!



VCA's controlled by alternate triangles provide automaic quad panning.

figure 11



Both Voltage and manually controlled Effects Processors are useful with Shepard Functions. Multiple outputs can be mixed for stereo use.

figure 12

LAB NOTES: Shepard Functions

Things I Forgot to Say
and other details

by
John Simonton

I regret not acknowledging "Shepard" as **Roger N Shepard** of Bell Telephone Labs. The paper in which he describes the Tone is titled "Circularity in Judgments of Relative Pitch" and appeared in THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA Vol 36, No 12 (December, 1964, I think).

I meant to acknowledge the encouragement of Don Slepian of Don and Judy Records.

Some of the figure numbers in the Polyphony article were scrambled and some were missing. They have been straightened out in the preceding reprint.

I forgot to mention that the peak values of the ramps and triangles is not the same. For any given setting of the

Range trimmer (R6), the peak value of the ramps is twice that of the triangles. Since the ramps and triangles will customarily be scaled either with attenuators or by the scaling controls of the driven device (range, depth, etc.), this should present no problems. Assuming the minimum recommended supply voltage of +/- 12v (+/- 18v max.) the maximum amplitude of the ramps is slightly more than 10v while that of the triangles is 5v. The circuitry will work with supply voltages as low as +/- 5v, but with greatly reduced function amplitude.

In the schematic diagram fig.8, the treatment of the ramp outputs Fr(4) - Fr(7) is meant to indicate that if the impedance of the control voltage input of the effect being used were high enough (greater than 1/2 meg.) buffers might not be needed if the value of the S/H capacitors (C19-C22 in this case) were increased. This is of no particular significance to you since the kit you purchased includes sufficient 4136 op-amps to buffer all 16 outputs. Accordingly, .01 mFd capacitors are included for use as C7-C22.

The switch S1 allows you to calibrate equipment. When pressed, it takes all 16 of the outputs to their ground potential, allowing the intial value of the oscillators, or effects to be set. Notice that because of offsets in the amplifiers there may be very slight differences in the "ground" voltage of each output and these should be compensated with the controls of the oscillator or effect.

For example, if you were going to generate a Shepard Tone using the equipment configuration show in Fig.9 of LAB NOTES, the calibration process would go something like this:

1) Tie all oscillator pitch control voltage inputs to one of the SFG Ramp outputs. Press the CAL button and adjust the INITIAL PITCH controls of the oscillators so that they all are at the same pitch.

2) Release the CAL button and adjust the RANGE controls of the oscillators so that they are in unison over about an 8 octave sweep (you will want to play with different RANGE settings to see the results).

3) Connect each oscillator to its own ramp and press the CAL button. Set the initial pitch control of all oscilators so that their output are the same (zero beat).

4) Release CAL and observe results.

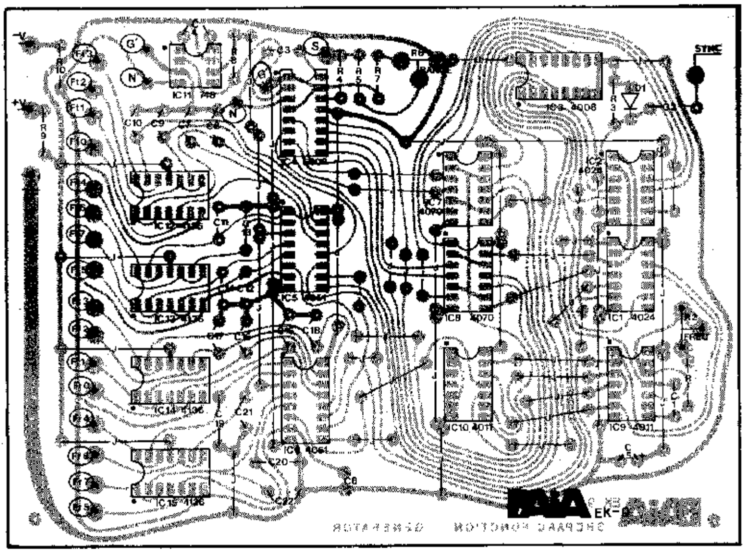
Note that the Shepard Function Generator RANGE trimmer can be used to vary the amplitude of all 16 functions simultaneously.

ASSEMBLY

Assembly of the experimenter's kit is straight forward. Clean the conductor side of the circuit board with steel wool or a Scotch-Brite type pad before installing resistors capacitors, sockets for IC's, trimmer resistors and diode. Parts placement designators are printed on the circuit board and duplicated in Fig.13 below. Observe polarity of Diode D1 and Electrolytic Capacitors C5 and C6.

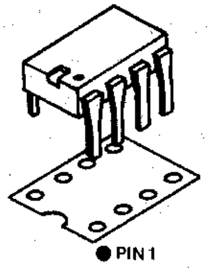
Using the bare wire provided, install the jumpers indicated by the "--J--" symbols printed on the circuit board. Cut the insulated wire provided into 2 equal length pieces and use to connect Circuit Board points G to G' and N to N'.

Complete assembly by inserting IC's in their sockets. CAUTION Most of these IC's are CMOS parts and can **be damaged** by discharges of static electricity. Do not wear **clothing** of synthetic material while handling these IC's. **Carefully** observe polarization of the IC's as indicated by **the pin 1** designation illustrated below.

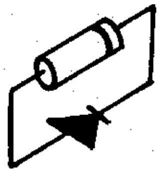


CIRCUIT BOARD PARTS PLACEMENT

figure 13



IC Polarity



Diode Polarity

EK-9 SHEPARD FUNCTION GENERATOR

Parts List

Part No	Value	Color Code
R1, R3, R8	10K	brown-black-orange
R4, R5	2700 ohms	red-violet-red
R7	1000 ohms	brown-black-red
R9, R10	100 ohms	brown-black-brown
R2	100K ohm Vertical Mount Trimmer (104)	
R6	50k ohm Vertical Mount Trimmer (503)	
C1	0.001 mFd disk	(102)
C2	0.05 mFd disk	(503)
C3, C7-C22	0.01 mFd disk	(103)
C4	15 pF disk	(15K)(15p)
C5, C6	100 mFd / 16v Electrolytic Capacitor	
D1	1N914 or 1N4148 diode	
IC1, IC2	(14 pin) 4024 7 Stage Counter	
IC3	(16 pin) 4008 4 Bit Full Adder	
IC4	(16 pin) 5008 8 Bit DAC	
IC5, IC6	(16 pin) 4051 1/8 Mux	
IC7, IC8	(14 pin) 4070 Quad Exclusive Or	
IC9, IC10	(14 pin) 4011 Quad Nand Gate	
IC11	(8 pin) 748 General Purpose Op-Amp	
IC12-IC15	(14 pin) 4136 Quad Op-Amp	

Miscellaneous kit parts:

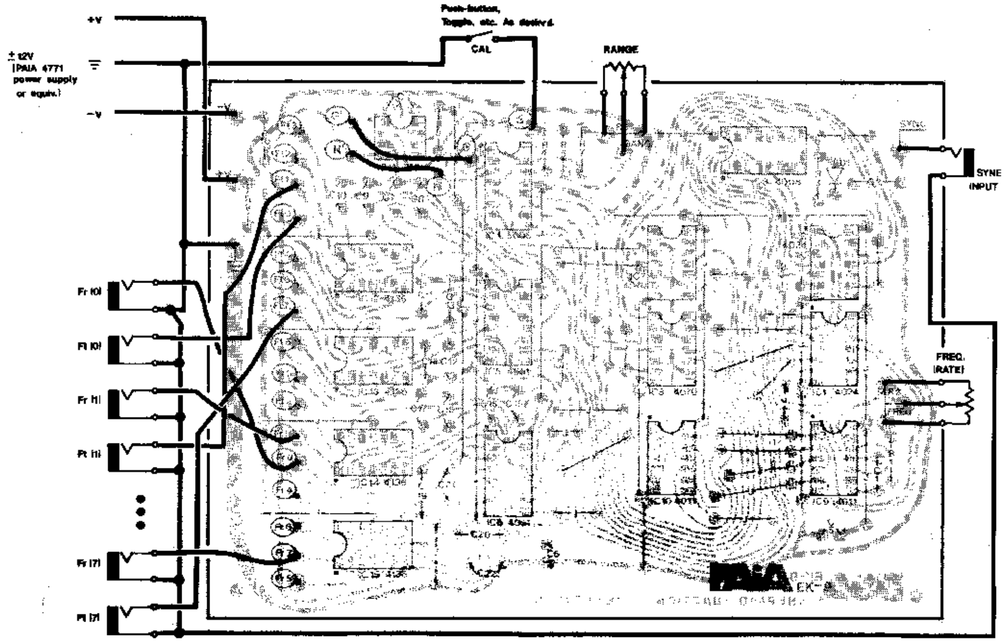
1 ea.	Shepard Function Generator Circuit Board
1 ea.	8 pin IC Socket
10 ea.	14 pin IC socket
4 ea.	16 pin IC socket
5 ft.	Bare Wire
6 in.	Stranded Insulated Wire

The completed Shepard Function Generator circuitry can be housed or mounted in variety of different ways, such as rack-panel mount or case enclosure.

Because of the high-level, low-impedance outputs, stranded insulated wire is appropriate for connections from circuit board to front panel jacks. Connectors can be chosen to be consistent with existing equipment. Possibilities include Phone Jacks, Phono Connectors, Pin Jacks or even DB Connectors.

Wiring from the circuit board to front panel jacks is illustrated in Fig.14, in the interest of clarity, not all connections are shown. Note in particular that the outputs on the edge of the PC board are not in sequence.

If desired, the PC Mount Trimmer resistors R2 and R6 can be removed and panel mount potentiometers of like value substituted in their place.



NOTICE THAT THE FUNCTION OUTPUTS ARE NOT IN SEQUENCE

figure 14

NOTES